

Chapter NO. 1

Notes

- About digits, numbers, number systems, Radix/Base
- Decimal and Binary Number Systems
- Decimal to Binary Conversion Table and vice versa
- Decimal to Binary Conversion
- Other conversions that follow above would be discussed in class.
- Binary Arithmetic (Addition, Subtraction, Multiplication and Division)

DIGIT AND NUMBERS

- **Digit:**

- A digit is a single numerical symbol used to represent numbers in a given number system.
- In the decimal system (base-10), digits range from 0 to 9.
- For example, in the number 456, each of the characters '4', '5', and '6' are digits.

- **Number:**

- A number is a mathematical concept used to represent a quantity, value, or position.
 - It can be composed of one or more digits, possibly including a decimal point and a sign.
 - Numbers can be integers (whole numbers) or real numbers (numbers with a fractional part).
- digits used in a number system. These concepts are fundamental to understanding how numbers are represented and manipulated in different contexts.

NUMBER SYSTEM AND RADIX

- **Number System:**

- A number system, also known as a numeral system or base system, is a mathematical notation for representing numbers using digits.
- Each number system has a base or radix, which determines the number of digits used and their positional value.
- The most common number systems include decimal (base-10), binary (base-2), octal (base-8), and hexadecimal (base-16).

- **Radix:**

- The radix, also known as the base, is the number of unique digits used in a particular number system.
 - It determines the value of each digit position in a number.
 - For example, in the decimal system (base-10), the radix is 10 because there are 10 unique digits (0 through 9).
 - In binary (base-2), the radix is 2 because there are only 2 unique digits (0 and 1).
 - Similarly, octal (base-8) has a radix of 8, and hexadecimal (base-16) has a radix of 16.
- a digit is a single numerical symbol, a number represents a quantity or value, a number system is a mathematical notation for representing numbers, and the radix is the base or number of unique how numbers are represented and manipulated in different contexts.

FOUR MAIN TYPES OF NUMBER SYSTEMS

- The four main types of number systems are:

- i. Binary Number System (Base-2):**

Binary is widely used in digital electronics and computer science.

- ii. Decimal Number System (Base-10):**

Decimal numbers are used in everyday life for counting, arithmetic, and representing quantities.

- iii. Octal Number System (Base-8):**

Octal numbers are commonly used in computer programming, especially for representing groups of three binary digits (bits).

- iv. Hexadecimal Number System (Base-16):**

Hexadecimal numbers are widely used in computer science, particularly in programming and digital communication, for representing binary data in a more compact and readable format.

BINARY NUMBER SYSTEM (BASE-2):

- **Binary Number System (Base-2):**
- The binary number system, also known as base-2, uses only two digits: 0 and 1.
- Each digit in a binary number represents a power of 2.
- It is commonly used in digital electronics and computer science to represent information using binary digits (bits).

OCTAL NUMBER SYSTEM (BASE-8):

- **Octal Number System (Base-8):**

- The octal number system, also known as base-8, uses eight digits: 0 to 7.
- Each digit in an octal number represents a power of 8.
- Octal numbers are often used in computer programming for representing groups of three binary digits (bits) compactly.

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DECIMAL NUMBER SYSTEM (BASE-10):

- **Decimal Number System (Base-10):**
- The decimal number system, also known as base-10, is the most familiar number system to humans.
- It uses ten digits: 0 to 9.
- Each digit in a decimal number represents a power of 10.
- Decimal numbers are used in everyday life for counting, arithmetic operations, and expressing quantities.
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HEXADECIMAL NUMBER SYSTEM (BASE-16):

- **Hexadecimal Number System (Base-16):**
 - The hexadecimal number system, also known as base-16, uses sixteen digits: 0 to 9 and A to F (representing values 10 to 15).
 - Each digit in a hexadecimal number represents a power of 16.
 - Hexadecimal numbers are commonly used in computer science, particularly in programming and digital communication, for representing binary data in a more compact and readable format

Number System

- A term used for a set of symbols/digits that represent various numerical values in the world.
- The total number of symbols/digits that comprise a number system is called its radix/base of such number system.
- There are different number systems which include decimal, binary, octal and hexadecimal number system.
- The decimal number system/natural number system is used in our daily life whereas the digital/electronic machines only understand binary number system.

Number System continue.....

- In every number system, the value has a place value, in decimal number system the place value is equal to ten raised to some power, from right to left. Alternately a number 345 in decimal number system can be written as:

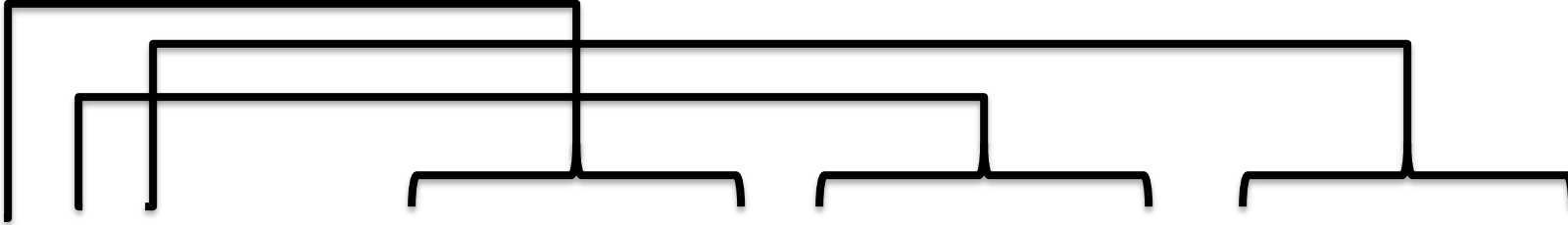
$$3*10^2 + 4*10^1 + 5*10^0 = 300 + 40 + 5 = 345$$

- In above three-digit number 3 carries the most weight of the three digits and is called the Most Significant Digit(MSD), whereas the 5 carries the least weight which is called the Least Significant Digit (LSD).
- In each number system the place value of a digit is obtained by multiplying the digit with the base raised to some integer power.

Decimal Number System

- It is the most commonly used number system.
- It is called decimal because its base is 10 hence it has 10 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).
- In decimal number system the maximum value of a single digit is 9 (one less than the value of the base i.e. 10)
- Each position of a digit represents a specific power of the base (10) starting with 10^0 from right side of the number

Example of Decimal Number System:



The diagram illustrates the expansion of the decimal number 689 into its place value components. Three horizontal brackets are drawn above the number. The top bracket spans the entire number '689'. The middle bracket spans the '89' part. The bottom bracket spans the '9' part. Vertical lines connect the ends of these brackets to the corresponding digits in the equation below.

$$\begin{aligned} 689_{10} &= (6 \times 10^2) + (8 \times 10^1) + (9 \times 10^0) \\ &= 6 \times 100 + 8 \times 10 + 9 \times 1 \\ &= 600 + 80 + 9 \end{aligned}$$

Binary Number System

- It is mostly used in computer applications.
- It is called binary because its base is 2 hence it has 2 symbols or digits (0, 1).
- In binary number system the maximum value of a single digit is 1 (one less than the value of the base i.e. 2).
- Each position of a digit represents a specific power of the base (2) starting with 2^0 from right side of the number.
- In a binary number, each **binary digit** is called as a bit. The leftmost binary digit(bit) is called Most Significant Bit (MSB) and the rightmost binary digit(bit) is called Least Significant Bit (LSB), respectively.

Example of Binary Number System:

Example

$$\begin{aligned}10101_2 &= (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= 16 + 0 + 4 + 0 + 1 \\ &= 21_{10}\end{aligned}$$

Octal Number System

- Every number in this number system is made up of eight symbols such as 0, 1, 2, 3, 4, 5, 6, 7.
- Its base is 8, each digit place value in this system is obtained by multiplying the digit with 8 raised to some power.
- The word octal means eight.
- the octal number 678 can be alternately expressed as:

$$678 = 6 * 8^2 + 7 * 8^1 + 8 * 8^0$$

Hexadecimal Number System

- Hexa means Six and deci means 10 so the sum is 16, so this number system used 16 symbols to represent different numerical values. The symbols/digit in this system are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.
- The base/radix of this system is 16. so the place value of each digit is obtained by multiplying such digit with 16 raised to some integer exponent.

COVERSIONS:

Decimal to Binary Table:

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1	1	0	0	0	1	0	0



$$128 + 64 + 0 + 0 + 0 + 4 + 0 + 0$$

Since computer uses binary notation and binary codes therefore , number/arithmetic in other system are carried out in binary and then processes accordingly.

Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Decimal to Binary Conversion:

- Successively divide number by new base(b), taking remainder as result.
- The remainders in the reverse order of their appearance will be the new base equivalence.
- Example: Convert 57_{10} to binary.

$$57 / 2 = 28 \text{ remainder } 1 \text{ (Least Significant Bit)}$$

$$/2 = 14 \text{ remainder } 0$$

$$/2 = 7 \text{ remainder } 0$$

$$/2 = 3 \text{ remainder } 1$$

$$/2 = 1 \text{ remainder } 1$$

Ans: 111001_2

$$/2 = 0 \text{ remainder } 1 \text{ (MSB)}$$

Three step process:

- 1) Divide the decimal number by 2 (base of Binary number system).
- 2) Repeat this method until the quotient is less than 2.
- 3) Bottom to top sequence of remainders will be the required binary number.

Convert 53 to Binary :

$$2 \overline{)53}$$

$$2 \overline{)26}$$

$$2 \overline{)13}$$

$$2 \overline{)6}$$

$$2 \overline{)3}$$

$$2 \overline{)1}$$

$$0$$

$$\text{rem.} = 1 = a_0$$

$$\text{rem.} = 0 = a_1$$

$$\text{rem.} = 1 = a_2$$

$$\text{rem.} = 0 = a_3$$

$$\text{rem.} = 1 = a_4$$

$$\text{rem.} = 1 = a_5$$

$$53_{10} = 110101_2$$

Convert 101_2 to Decimal :

$$\begin{aligned}101_2 &= (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) \\ &= 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 4 + 0 + 1 \\ &= 5_{10}\end{aligned}$$

Some other conversions

- 1) Decimal to octal and vice versa
- 2) Decimal to hexadecimal and vice versa
- 3) Octal to binary and vice versa (NOTE: each octal digit is equal to three bit binary number so just replace each octal digit with three binary digits, starting from the right most octal digit.)
- 4) Hexadecimal to binary and vice versa (NOTE: each hexadecimal digit is equal to four bit binary number so just replace each hexadecimal number digit with four binary digits, starting from the right most hexadecimal digit.)

Binary Arithmetic

- Binary arithmetic has same operations as that of decimal arithmetic, the only difference is the total number of symbols /digits available for use in the system.
- Binary arithmetic is essential in all digital computers and in many other types of digital systems.
- All Binary arithmetics are same as Decimal arithmetics.

Binary Arithmetic

Binary Addition:

The four basic rules for adding binary digits are as follows:

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10 \text{ (zero with a carry of 1)}$$

For three binary digits Rules:

$$1+1+1 = 3 = 11 \text{ (1 with a carry of 1)}$$

$$1+0+1 = 10 \text{ 0 with a carry of 1.}$$

$$1+1+0 = 10 \text{ , 0 with a carry of 1.}$$

$$1+0+0 = 1$$

Or

$$1 + 1 = 0 \text{ plus a carry of 1 to next left column}$$

Binary Arithmetic

Binary Addition Example:

	Binary	Decimal
Carry	1 1	
	1 0 0 1 1 1	39
	+ 1 1 0 0 1 0	
	-----	+ 50
	1 0 1 1 0 0 1	-----
		89

Binary Arithmetic

Binary Subtraction:

The four basic rules for subtracting binary digits are as follows:

$$0 - 0 = 0$$

$$0 - 1 = 1 \text{ with a borrow from the next column}$$

$$1 - 0 = 1$$

$$1 - 1 = 0$$

Binary Arithmetic

Binary Subtraction Example:

$$\begin{array}{r} \text{Borrow} \qquad \qquad \qquad 1 \qquad 1 \\ \qquad \qquad \qquad 10 \rightarrow 10 \rightarrow 10 \\ \begin{array}{r} 1 \ 0 \ 0 \ 0 \\ - \qquad \qquad 1 \ 1 \\ \hline 1 \ 0 \ 1 \end{array} \end{array}$$

Binary Arithmetic

Binary Multiplication:

The four basic rules for multiplying binary digits are as follows:

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

$$\begin{array}{r} 1010 \\ \times 110 \\ \hline 0000 \\ 1010 \\ 1010 \\ \hline 111100 \end{array}$$

Binary Arithmetic

Binary Division:

The four basic rules for dividing binary digits are as follows:

$0 / 0 = \text{error (cannot divide by zero)}$

$0 / 1 = 0$

$1 / 0 = \text{error (cannot divide by zero)}$

$1 / 1 = 1$

Binary Arithmetic

Binary Division Example:

The diagram illustrates the binary division of 111011 by 11. The dividend is 111011 and the divisor is 11. The quotient is 10011. The process is shown in several steps:

- Step 1: 111011 divided by 11. The first two 1s of the dividend are subtracted from the divisor, resulting in 00101.
- Step 2: The next 1 of the dividend is brought down, forming 00101. The divisor 11 is subtracted from 101, resulting in 101.
- Step 3: The next 1 of the dividend is brought down, forming 101. The divisor 11 is subtracted from 101, resulting in 10.

Annotations include blue arrows pointing from the dividend digits to the subtraction steps, and blue dashed lines connecting the quotient digits to the corresponding subtraction steps.

$$\begin{array}{r} 111011 \quad | \quad 11 \\ \underline{11} \\ 00101 \\ \underline{11} \\ 101 \\ \underline{11} \\ 10 \end{array}$$